Question 7

Is it possible for two vectors of different magnitudes to add to zero? Is it possible for three vectors of different magnitudes to add to zero? Explain.

Solution

It's not possible for two vectors of different magnitudes to add to zero because they can't be antiparallel. And if they're not antiparallel, there will be a nonzero component perpendicular to one of the vectors. This is a geometric argument. For an algebraic argument, consider two vectors, $\overrightarrow{\mathbf{A}} = \langle A_x, A_y \rangle$ and $\overrightarrow{\mathbf{B}} = \langle B_x, B_y \rangle$. For them to add to zero, their components would have to add to zero.

$$\begin{cases} A_x + B_x = 0 \\ A_y + B_y = 0 \end{cases}$$

Solve for B_x and B_y.

$$\begin{cases} B_x = -A_x \\ B_y = -A_y \end{cases}$$

As a result, the two vectors can add to zero as long as $\overrightarrow{\mathbf{A}} = \langle A_x, A_y \rangle$ and $\overrightarrow{\mathbf{B}} = \langle -A_x, -A_y \rangle$. But these two vectors have the same magnitude, so it's not possible. Suppose now that there's a third vector $\overrightarrow{\mathbf{C}} = \langle C_x, C_y \rangle$. For them to add to zero, their components would have to add to zero.

$$\begin{cases} A_x + B_x + C_x = 0 \\ A_y + B_y + C_y = 0 \end{cases}$$

Solve for C_x and C_y.

$$\begin{cases} C_x = -A_x - B_x \\ C_y = -A_y - B_y \end{cases}$$

As a result, the three vectors can add to zero as long as $\overrightarrow{\mathbf{A}} = \langle A_x, A_y \rangle$ and $\overrightarrow{\mathbf{B}} = \langle B_x, B_y \rangle$ and $\overrightarrow{\mathbf{C}} = \langle -A_x - B_x, -A_y - B_y \rangle$. So it is possible for three vectors to add to zero while having different magnitudes.